Generation of two-mode entanglement between separated cavities

Pengbo Li

State Key Laboratory for Mesoscopic Physics, Peking University, Beijing 100871, China
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We propose a scheme for the generation of two-mode entangled states between two spatially separated cavities. It utilizes a two-level atom sequentially coupling to two high-Q cavities with a strong classical driving field. It is shown that by suitably choosing the intensities and detunings of the fields and coherent control of the dynamics, several different entangled states such as entangled coherent states and Bell states can be produced between the modes of the two cavities.

The preparation of quantum entangled states continues attracting intense theoretical and experimental activities. These nonclassical states not only are utilized to test fundamental quantum mechanical principles such as Bell's inequalities [1] but also plays a central role in practical applications of quantum information processing [2] such as quantum computation[3, 4], quantum teleportation[5], and quantum cryptography[6]. In quantum optics the generation of various nonclassical states especially entangled states of electromagnetic fields is a central topic [7]. In the context of cavity QED [8, 9, 10], experimental realizations of the entanglement between two different modes sharing a single photon or two polarized photons in a cavity have been reported [11, 12]. In recent years, great effort has been put into preparation of the Schrödinger cat states [13, 14], where the extreme cat states are reduced to mesoscopic quantum states with classical counterparts, i.e., coherent states. Recently, a method of generating entangled coherent states [15] between different modes in a cavity has been presented [16]. There are also several other proposals for producing entangled field states between different cavities [17, 18, 19, 20, 21].

In this paper, we propose a scheme for generating two-mode entangled states such as entangled coherent states and Bell states between two distant cavities. A two-level atom is sent sequentially into two spatially separated cavities, assisted by a strong classical driving field [16]. We demonstrate that by suitably choosing the intensities and detunings of the fields and coherent control of the dynamics, several different entangled states such as entangled coherent states and Bell states can be produced between the two cavity modes. These entangled states of fields can have both fundamental applications in quantum mechanics and practical applications in quantum information processing. With presently available experimental setups in cavity QED, this protocol can be implemented.

This proposal consists of two distant high-Q cavities and a two-level atom, as sketched in Fig. 1. The atom sequentially couples to the cavities A and B (with cavity modes of frequencies of ν_A and ν_B), driving by a strong classical field (frequency ω_L). The ground state of the atom is labeled as $|g\rangle$, and the excited state as $|e\rangle$. In each cavity, the transition $|g\rangle \leftrightarrow |e\rangle$ (transition frequency ω_0) is coupled by the cavity mode with the coupling constants g_A and g_B, respectively. Furthermore, a strong classical
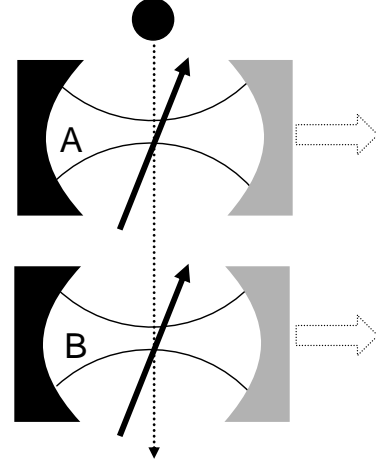

FIG. 1: Proposed experimental setup. A two-level atom sequentially couples to two distant cavities A and B, driving by a strong classical field.

field drives the same transition with a Rabi frequency $\Omega_A(\Omega_B)$ in each step. The associated Hamiltonian for the dynamics in each cavity under the dipole and rotating wave approximation is given by (let $\hbar = 1$)

$$
\begin{aligned}
H_j = \quad & \omega_0 \sigma^\dagger \sigma + \nu_j \hat{a}_j^\dagger \hat{a}_j \\
& + \Omega_j(e^{-i\omega_L t}\sigma^\dagger + e^{i\omega_L t}\sigma) \\
& + g_j(\sigma^\dagger \hat{a}_j + \sigma \hat{a}_j^\dagger), (j = A, B). \quad (1)
\end{aligned}
$$

Where $\sigma = |g\rangle\langle e|$, and $\sigma^\dagger = |e\rangle\langle g|$ are the atomic transition operators; $\hat{a}_j$ and $\hat{a}_j^\dagger$ are the annihilation and creation operators with respect to cavity j. In the following we assume that Ω_A, Ω_B, g_A, and g_B are real for simplicity.

The Hamiltonian of Eq. (1) can be changed to a reference frame rotating with the driving field frequency ω_L,

$$
\begin{aligned}
H_j^L = \quad & \Delta\sigma^\dagger \sigma + \delta_j \hat{a}_j^\dagger \hat{a}_j + \Omega_j(\sigma^\dagger + \sigma) \\
& + g_j(\sigma^\dagger \hat{a}_j + \sigma \hat{a}_j^\dagger), (j = A, B), \quad (2)
\end{aligned}
$$

with $\Delta = \omega_0 - \omega_L$ and $\delta_j = \nu_j - \omega_L$. In the following we will set $\Delta = 0$ for simplicity. We now switch to a new atomic basis $|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}$. In the interaction

picture with respect to $H_0^j = \delta_j \hat{a}^\dagger \hat{a}_j + \Omega_j(\sigma^\dagger + \sigma)$, we have the following Hamiltonian

$$H_j^I = \frac{g_j}{2}(|+\rangle\langle+| - |-\rangle\langle-| + e^{2i\Omega_j t}|+\rangle\langle-|$$
$$- e^{-2i\Omega_j t}|-\rangle\langle+|)\hat{a}_j e^{-i\delta_j t} + \text{H.c.} \qquad (3)$$

The Hamiltonian (3) is the starting point in the following discussions, from which we show that a variety of entangled states of the two distant cavities can be generated through suitably choosing the detunings and light intensities.

We first show how to produce the entangled coherent states between the two cavities. Assume that $\Omega_A(\Omega_B) \gg \{g_A, \delta_A(g_B, \delta_B)\}$, this strong driving condition can allow one to realize a rotating-wave approximation and neglect the fast oscillating terms. Now H_j^I reduces to

$$H_j^I = \frac{g_j}{2}(|+\rangle\langle+| - |-\rangle\langle-|)(\hat{a}_j e^{-i\delta_j t} + \hat{a}_j^\dagger e^{i\delta_j t})$$
$$= \frac{g_j}{2}(\sigma^\dagger + \sigma)(\hat{a}_j e^{-i\delta_j t} + \hat{a}_j^\dagger e^{i\delta_j t}). \qquad (4)$$

If we choose $\delta_j = 0$, this Hamiltonian corresponds to the simultaneous realization of Jaynes-Cummings (JC) [22] and anti-Jaynes-Cummings (AJC) interaction in each cavity. The evolution operator for the system is given by

$$U_j(t) = e^{-iH_j^I t}$$
$$= e^{-\frac{ig_j t}{2}(\sigma^\dagger + \sigma)(\hat{a}_j + \hat{a}_j^\dagger)}$$
$$= \hat{D}(\alpha_j)|+\rangle\langle+| + \hat{D}(-\alpha_j)|-\rangle\langle-|, \qquad (5)$$

with $\hat{D}(\alpha_j) = e^{\alpha_j \hat{a}_j^\dagger - \alpha_j^* \hat{a}_j}$, and $\alpha_j = -ig_j t/2$. Assume that at the time $t = 0$ the system is prepared in the ground state $|g\rangle|0\rangle_A|0\rangle_B = (|+\rangle + |-\rangle)|0\rangle_A|0\rangle_B/\sqrt{2}$, i.e., the atom stays in $|g\rangle$, and the two cavities are in the vacuum states. The atom enters cavity A and undergoes the dynamics of Eq. (5). The evolved state after a time t_A will be

$$\frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_A + |-\rangle|-\alpha\rangle_A)|0\rangle_B, \qquad (6)$$

where $\alpha = -ig_A t_A/2$ for the case of $\delta_A = 0$. This microscopic-mesoscopic entangled state is the so-called Schrödinger cat state. After an interaction time t_A in cavity A, the atom enters cavity B. It will also undergoes the dynamics of Eq. (5). After a time t_B, the system consisting of the two cavities and the atom will evolve into

$$\frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_A|\beta\rangle_B + |-\rangle|-\alpha\rangle_A|-\beta\rangle_B), \qquad (7)$$

with $\beta = -ig_B t_B/2$ for the case of $\delta_B = 0$. Equation (7) describes a tripartite entangled state involving one microscopic and two mesoscopic systems. If we measure the atomic state in the bare basis $\{|g\rangle, |e\rangle\}$, we can obtain the entangled coherent states of the fields in the interaction picture

$$\mathcal{N}_{AB}^\pm(|\alpha\rangle_A|\beta\rangle_B \pm |-\alpha\rangle_A|-\beta\rangle_B), \qquad (8)$$

where $\mathcal{N}_{AB}^\pm$ is the normalized factor. It has been shown that these states can be utilized as an important tool in the field of quantum information. Here we propose to generate the entangled coherent states between two distant resonators.

We next show that using the interaction described by Eq. (3) one can generate the maximally entangled state $1/\sqrt{2}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)$ between the two cavities. If we choose $\delta_j = 2\Omega_j$ and $|\delta_j| \gg g_j$, then we can bring the Hamiltonian (3) to the JC interaction in the $|\pm\rangle$ atomic dressed basis

$$H_j^{JC} = \frac{g_j}{2}(|+\rangle\langle-|\hat{a}_j + |-\rangle\langle+|\hat{a}_j^\dagger). \qquad (9)$$

Assume at $t = 0$, the system stays in $|+\rangle|0\rangle_A|0\rangle_B$. Then after an interaction time $t_A = \pi/(2g_A)$ in cavity A, the system will evolve into

$$\frac{1}{\sqrt{2}}(|+\rangle|0\rangle_A - i|-\rangle|1\rangle_A)|0\rangle_B. \qquad (10)$$

Subsequently, the atom enters cavity B, and undergoes the dynamics of Eq. (9). Then the atom-field state will be

$$\frac{1}{\sqrt{2}}(\cos(g_B t/2)|+\rangle|0\rangle_B - i\sin(g_B t/2)|-\rangle|1\rangle_B)|0\rangle_A - \frac{i}{\sqrt{2}}|-\rangle|1\rangle_A|0\rangle_B. \qquad (11)$$

If the interaction time $t_B = \pi/g_B$ is taken, the final state will be

$$\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|-\rangle, \qquad (12)$$

where a common phase factor $-i$ has been discarded. Clearly, at this time the atomic state has been factorized out and the modes of two distant cavities end up in the

EPR pair state[23]

$$\frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B). \qquad (13)$$

From the viewpoint of quantum information, this is a maximally entangled state of two qubits stored in the modes of two distant cavities.

Now we consider the case of generating the entangled state $1/\sqrt{2}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$ between the two cavities. If we choose $\delta_A = 2\Omega_A$ and $|\delta_A| \gg g_A$, then we can realize the JC interaction in cavity A, which is described by Eq. (9). However, in cavity B, we need the AJC interaction, which requires the relation $\delta_B = -2\Omega_B$ and $|\delta_B| \gg g_B$ be chosen. Then we can bring the Hamiltonian (3) to the AJC interaction in the $|\pm\rangle$ atomic dressed basis

$$H_B^{AJC} = \frac{g_B}{2}(|-\rangle\langle+|\hat{a}_B + |+\rangle\langle-|\hat{a}_B^\dagger). \qquad (14)$$

The brief idea of producing the target entangled state is as follows. We send a Rydberg atom preparing in state $|+\rangle$ into cavity A and make it undergo the dynamics governed by the JC interaction. After an interaction time $t_A = \pi/(2g_A)$, the atom is allowed to enter cavity B, undergoing the dynamics governed by the AJC interaction of Eq. (14). If we take $t_B = \pi/g_B$, the final state for the cavity modes will be $1/\sqrt{2}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)$.

In order to illustrate the idea explicitly, we employ the time evolution operator approach. Assume that at $t = 0$, the system is in state $|+\rangle|0\rangle_A|0\rangle_B$. At the stage of the atom interacting with cavity A, the system is governed by the interaction of Eq. (9). Then after an interaction time $t_A = \pi/(2g_A)$ in cavity A, the atom-field state will be the same as equation (10). Subsequently, the atom enters cavity B, and undergoes the dynamics of Eq. (14). Then the atom-field state will be

$$\frac{1}{\sqrt{2}}|-\rangle|0\rangle_A|0\rangle_B - \frac{i}{\sqrt{2}}(\cos(g_Bt/2)|-\rangle|0\rangle_B - i\sin(g_Bt/2)|+\rangle|1\rangle_B)|1\rangle_A. \qquad (15)$$

If the interaction time $t_B = \pi/g_B$ is taken, the final state will be

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B)|+\rangle. \qquad (16)$$

The atomic state has been factorized out and the modes of two distant cavities end up in the following entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B). \qquad (17)$$

This state is also a maximally entangled state. Together with state (13), they form two of the well-known Bell states. These states have both fundamental applications in quantum mechanics and practical applications in quantum information processing.

It is necessary to analyze the proposal requirements. To realize this scheme, a two-level atom needs to sequentially interact with two distant cavities. To generate the entangled coherent states of two cavity modes, the initial states of the atom and two cavities have only to be the ground states. After the atom leaves cavity B, a tripartite entangled state involving one microscopic and two mesoscopic systems has been prepared. One needs to measure the atomic state in the bare basis to obtain the entangled coherent states between two cavities. Therefore, to realize this protocol requires that the atom and two cavity modes should not decay during this process. One can control the interaction time of the atom with the two cavities in experiments and implement this proposal in the strong coupling regime to meet the requirements.

In fact, from the expressions for the parameters α and β, we know that the total time for preparing the target entangled states is determined by both the interaction time t_A, t_B and the coupling strengths g_A, g_B. In the case of generating Bell states, the preparation time should be within the decay time of the atom and the two cavities as well. However, in this case the interaction time (the preparation time) is related to the coupling strengths. One needs to suitably control the interaction time to implement this protocol. Another requirement for producing Bell states is at the beginning of the experiment, the atom needs to be prepared in the coherent superposition state $|+\rangle = 1/\sqrt{2}(|g\rangle + |e\rangle)$, which can be produced by applying a $\pi/2$ pulse to the atom before entering the cavity [9].

We consider some experimental matters. For a potential experimental system and set of parameters in microwave resonators [9], the atomic configuration could be realized in Rydberg atoms. We choose the single photon dipole coupling strength as $g_A \sim g_B = g/(2\pi) \sim 50$ kHz [9]. Then for generating entangled coherent states, the preparation time is about $T \sim 0.06$ ms with averaged photon number $|\alpha| = |\beta| = 5$ in each cavity. In the case of producing Bell states, the preparation time is about $T \sim 0.02$ ms. These results are in line with the current experimental setups. Resonators stable over 100 ms have been reported recently [24]. The radiative life time for Rydberg atoms is about $T_a \sim 30$ ms [9]. Therefore, the time needed to complete the procedure is much shorter than the decay time of the atom and the cavities.

In conclusion, we have proposed a scheme for the gen-

eration of two-mode entangled states between two separated cavities. It relies on a two-level atom sequentially coupling to two high-Q cavities with a strong classical driving field. We demonstrate that by suitably choosing the intensities and detunings of the fields and coherent control of the dynamics, several different entangled states such as entangled coherent states and Bell states can be produced between the two cavity modes. With presently available experimental setups in cavity QED, the realization of this proposal is feasible.

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